1617-257 Wed Sep 21, Hour 5: Topology in R^n, cont.
cont.

September 18, 2016 3:11 PM

Post signs

on board

All about: http://drorbn.net/1617-257
Wiki riddle repository?

Riddle Along: on any pair of potatos, you can draw a pair of 3D-congruent curves. T/F?

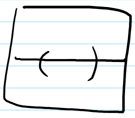
Read Along: Munkres sec. 3.

open sets: $\emptyset, X$, arbitrary unions, finite intersections

closed sets: $\emptyset, X$, arbitrary intersections, finite unions.

on $\mathbb{R}^n$, both notions are same for $\|\cdot\|$ as for $|\cdot|$.

We know how to reduce both to subspaces.

Some examples: 

Def Limit x_0 of $A \subset X$:

$$\forall \epsilon > 0 \quad \left(U(x_0,\epsilon) \setminus \{x_0\}\right) \cap A \neq \emptyset$$

Equiv: every nbd of x_0 contains ∞-many elements of A.

closure: $\bar{A} = A \cup \{\text{limit pts of } A\}$

Thm A is closed $\Leftrightarrow A = \bar{A}$

Pf really, A^c open $\Leftrightarrow lpA \subset A$

$\Rightarrow$ Suppose $x \in lpA \setminus A \ldots$

$\Leftarrow$ suppose $x \in A^c$; if is not a l.p., so....

Ex: 1. $\bar{A}$ is the "smallest" closed set containing A.
2. $\bar{A}$ is the intersection of all closed sets containing A.

Suppose X & Y are metric, w/ metrics d_X & d_Y

Def $f: X \to Y$ is cont. at $x_0 \in X$ if for every nbd V of $f(x_0)$ [:= an open set containing $f(x_0)$] there is a nbd U of x_0 s.t. $f(U) \subset V$

$\Leftrightarrow$
$$\forall \epsilon > 0 \; \exists \delta > 0 \quad d_X(x, x_0) < \delta \Rightarrow d_Y(f(x), f(x_0)) < \epsilon$$

Def $f: X \to Y$ is cont. means $\forall x_0 \in X$, f is cont. at x_0.

Thm TFAE for $f: X \to Y$:

1. f is continuous.

down line

2. For every V open in Y, $f^{-1}(V)$ is open in X

3 1 2

3 2. For every V open in Y, $f^{-1}(V)$ is open in X 1 line
3. For every F closed in Y, $f^{-1}(F)$ is closed in X 2
4 if $X=Y=\mathbb{R}$, f is cont. in the 157 sense. 4 done

<u>Thm</u> 1. constant functions are continuous.

2. $I: X \to X$ is cont.

3. $f: X \to Y$ cont, $A \subset X \Rightarrow f|_A : A \to Y$ is cont.

4. $f: X \to Y$, $g: Y \to Z \Rightarrow f/\!/g = g \circ f$ is cont.

5. $f: X \to \mathbb{R}^n$ is $(f_1(x), f_2(x) \dots f_n(x))$.
Then f is cont $\Leftrightarrow \forall i$ f_i is cont.

6. $f, g: X \to \mathbb{R}$ cont $\Rightarrow f+g$, $f \cdot g$, $f-g$, $\frac{f}{g}$ (where defined) cont.

7. $\pi_i : \mathbb{R}^n \to \mathbb{R}$ is cont.

Example: $f: \mathbb{R} \to \mathbb{R}$ by $x \longmapsto x^x$ is cont.

Skipped (but required): $f: X \to Y$, $\lim_{x \to x_0} f(x)$
.... some theorems ....

<u>Def</u> $\text{int}\, A =$ union of all open sets contained in A $= \left\{ x \in A : \begin{array}{l} \exists \epsilon > 0 \\ U(x, \epsilon) \subset A \end{array} \right\}$

$\text{Ext}\, A = \text{int}\, A^c =$ union of all open sets disjoint from A.

$\text{Bd}\, A = X \setminus (\text{int}\, A \cup \text{ext}\, A)$

<u>Claim</u> $\text{int}\, A = \overline{(X \setminus A)}$, $\text{Ext}\, A = X \setminus \overline{A}$
$\text{Bd}\, A = \overline{A} \cap \overline{(X \setminus A)}$

HW: Read the rest of section 3, about limits, interiors, exteriors.